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MODELING THE VOLATILITY OF RETURNS ON INVESTMENT UNITS OF VOLUNTARY PENSION FUNDS IN SERBIA

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Abstract: The purpose of this paper is to model and analyze the volatility of returns on investment units in voluntary pension funds in Serbia, focusing on five funds: Dunav, Generali Basic, Generali Index, DDOR Garant Ekvilibrio, and Raiffeisen Future. Given the growing significance of voluntary pension funds, the study explores the role of investment units as a crucial financial instrument that allows diversification and optimization of long-term returns. Methodologically, the study applies the Extreme Value Theory (EVT) using the Generalized Pareto Distribution (GPD) to model the extreme events in the distribution tails, a key component for risk management. The ARCH test was used initially to assess heteroskedasticity in the time series, but the absence of significant volatility changes negated the application of GARCH models. Instead, EVT was implemented to capture rare, yet impactful, fluctuations. Additionally, Value at Risk (VaR) and Expected Shortfall (ES) were estimated based on the fitted GPD model, offering more robust risk quantification for extreme losses. The results indicate that all return series are highly correlated, with extreme values predominantly occurring in shorter bursts. GPD models successfully captured these extremes, and VaR and ES measures highlighted the periods of elevated risk, particularly during financial crises. This research presents an original contribution to the analysis of investment unit volatility, providing practical insights for fund managers in balancing risk and return in a volatile market environment.

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JEL classification: G11, G32, C53, C58, G17

1. Introduction

Voluntary pension funds are becoming an increasingly important mechanism for saving and investing in modern financial systems, especially in countries facing the challenges regarding the sustainability of public pension frameworks. Individuals are gradually relying on private savings to ensure an adequate income level during retirement, and voluntary pension funds provide flexible investment options that allow participants to choose between different investment units based on their risk and return preferences. Investment units serve as the primary instrument of these funds, enabling participants to allocate resources to various asset classes, including stocks, bonds, and money market funds, thereby facilitating risk diversification and potentially increasing long-term returns.

In the Republic of Serbia, two pension pillars currently operate: the mandatory state pension pillar (the first pillar) and the voluntary private pension pillar (the third pillar). When voluntary pension funds were introduced, it was believed that the state system would be significantly relieved, and accumulated savings would allow many to have a secure retirement (Radojković & Stevanović, 2024). The “Pay as you go” first pillar is a compulsory pension system. The second pillar is a mixed system where the employer pays pension contributions, with a percentage determined by the state being allocated to a voluntary insurance fund. The employee chooses the fund where the employer pays contributions. However, this pillar does not operate in Serbia. Voluntary pension funds function as a fully funded system, the third pillar, often referred to as a capital accumulation system or capitalized fund system. The amount of private pension depends on the accumulated funds in the member’s personal account and the returns on the invested funds (Kočović et al., 2010).

The role of investment units in voluntary pension funds is complex. They primarily enable pension funds to skillfully manage members' assets and allocate capital according to various investment strategies. Each investment unit has unique characteristics, including risk profiles, expected returns, and investment timeframes, making it easier for fund members to tailor their saving strategies to meet individual goals and requirements. For example, younger participants with an extended investment horizon may favor units with higher exposure to equities, offering higher returns but with increased volatility, while older participants may opt for more conservative units that focus on bonds or money market funds, reducing the risk of short-term capital depreciation.

To increase the transparency of voluntary pension funds and improve the comparability of investment unit value trends, the National Bank of Serbia introduced the unique FONDex index to track trends within the voluntary pension fund system (https://www.nbs.rs/sr_RS/finansijske-institucije/penzijski-fondovi/fondex/).

The main challenge in managing investment units within a voluntary pension fund lies in balancing returns and risk. To achieve optimal long-term returns, funds must efficiently diversify their investments and continuously assess the prevailing market conditions. Variations in global and local economic landscapes, capital market fluctuations, and regulatory changes can significantly impact the performance of different investment units. Therefore, it is imperative that fund managers proactively adjust investment strategies to maximize returns while minimizing participants' risk exposure.

Evaluating the performance of investment units within voluntary pension funds is crucial for assessing the overall efficiency of these funds. Monitoring metrics such as investment return, volatility, and risk management provides a deeper understanding of how different investment strategies contribute to fund objectives. Furthermore, detailed performance evaluation can offer valuable insights to participants on optimizing their investment choices in line with prevailing market conditions and personal financial goals.

Financial time series analysis plays a crucial role in risk management and predicting market trends, particularly in terms of volatility and extreme events that can significantly affect financial portfolios. Traditional models for volatility analysis, such as ARCH (Autoregressive Conditional Heteroskedasticity) and its extended versions, provide insights into the serial dependence of variance in the data, enabling the effective modeling of volatility over time. The ARCH test is a standard statistical tool used to test for the presence of heteroskedasticity in time series, identifying the potential need for models like GARCH or similar. However, standard models often fail to fully capture rare but significant extreme events that occur in markets, such as financial crises or large swings in stock values. In this context, Extreme Value Theory (EVT) offers a robust framework for assessing risk associated with these extreme events.

Applying EVT, with the use of the Generalized Pareto Distribution (GPD), allows for more precise modeling of return distribution tails, especially in assessing tail risks such as Value-at-Risk (VaR) and Expected Shortfall (ES). VaR is a key metric that quantifies the maximum expected loss for a given confidence level over a specified time period, while ES represents the average loss in cases where the VaR threshold is exceeded. The introduction of GPD within EVT enables the modeling of extreme events by accurately fitting the distribution of threshold exceedances, significantly improving VaR and ES estimation.

Additionally, estimating volatility in time series using the rolling standard deviation methodology provides a dynamic approach to tracking volatility changes over time. This technique uses a moving window to continuously update the standard deviation estimate based on the most recent data, offering a more accurate insight into current market volatility. By using rolling standard deviation in combination with VaR and ES estimates based on the fitted GPD distribution, it is possible to better identify periods of high volatility and adequately quantify risk during such periods.

This paper aims to integrate several approaches in the analysis of financial time series, including the ARCH test for testing heteroskedasticity, Extreme Value Theory (EVT) with Generalized Pareto Distribution (GPD) for modeling tail distribution and risk assessment, as well as the use of rolling standard deviation for dynamic volatility tracking. Together, these tools provide a comprehensive understanding of the risks and volatility in financial time series, offering a solid foundation for risk management and investment decision-making.

The structure of the paper is organized in a away that after the introductory part, the second part describes the methodology used in detail. This section covers statistical techniques such as the ARCH test, Extreme Value Theory (EVT), and modeling the distribution using the Generalized Pareto Distribution (GPD), along with methods for assessing Value-at-Risk (VaR) and Expected Shortfall (ES). The focus of this section is on the theoretical foundation and practical steps in applying these methods to the observed time series of investment unit returns. The third section of the paper presents the research results, including volatility analyses of investment unit returns, risk assessments based on GPD, and the interpretation of the obtained VaR and ES values. The results are presented through charts and tables, with a discussion highlighting key findings and their significance for modeling the volatility of investment units in voluntary pension funds in Serbia.

2. Research methodology

Modeling volatility in financial time series represents a complex challenge due to the specific characteristics of financial data, such as the presence of "fat tails" and the phenomenon of volatility clustering. Various methodologies have been developed to address these complexities. One prominent strategy involves applying Extreme Value Theory (EVT) to characterize the tails of distributions, which is particularly useful for capturing extreme fluctuations in financial markets. This methodology can be modified to incorporate non-stationarities, such as regime shifts, which often occur in financial time series during periods of market turmoil (Chavez-Demoulin et al., 2014).

GARCH (Generalized Autoregressive Conditional Heteroskedasticity) models are widely used in financial time series analysis to capture volatility clustering, a common phenomenon where periods of high volatility are followed by further high volatility, and periods of low volatility follow low volatility. The GARCH (1,1) model, a popular variant, is known for its ability to model fat-tailed distributions, which are frequently observed in financial returns (Haan et al., 2016). This model assumes that the conditional variance of returns is a function of past squared returns and past variances, allowing it to effectively model the time-varying volatility present in financial markets (Le, 2020). Empirical studies have shown that GARCH models, when combined with Extreme Value Theory (EVT), provide more accurate Value at Risk (VaR) and Expected Shortfall (ES) forecasts by focusing on the tail behavior of the return distribution, which is crucial for risk management (Le, 2020; Paul & Sharma, 2021). The integration of EVT with GARCH models enhances the model's ability to handle extreme market movements by fitting a Generalized Pareto Distribution (GPD) to standardized residuals that exceed a certain threshold (McNeil & Frey, 2000; Le, 2020). This approach has shown better performance compared to standalone GARCH models in forecasting, especially in emerging markets where extreme events are more frequent (Paul & Sharma, 2021). The ability of GARCH models to adapt to changing market conditions and their versatility in modeling complex financial time series make them a fundamental tool in the field of financial econometrics (Lux et al., 2016).

Extreme Value Theory (EVT) is a statistical approach that deals with the analysis of rare and extreme events, particularly at the tails of data distributions. It is used in fields such as finance, engineering, and meteorology, where it is essential to assess the probability of rare but significant occurrences, such as natural disasters or financial collapses. EVT is applied by identifying block maxima or threshold exceedances in financial data, which are then used to model the tail behavior of the distribution (Magnou, 2017). This approach helps calculate tail risk measures and their confidence intervals, providing a statistical basis for risk management decision-making (Gilli & Kellezi, 2006). Furthermore, EVT is used in various methodologies for estimating profit and loss distributions in financial portfolios, including non-parametric historical simulations and parametric models like GARCH, which assume conditional normality (McNeil & Frey, 2000). However, EVT offers a more reliable alternative because it does not rely on the assumption of normality, which often does not hold for real financial data (McNeil & Frey, 2000). By focusing on extremes, EVT allows financial institutions to better prepare for rare but potentially catastrophic market events, thereby playing a key role in developing more resilient financial risk management strategies (McNeil & Frey, 2000; Gilli & Kellezi, 2006).

One of the key models of EVT is the Generalized Pareto Distribution (GPD), which describes extreme values above a certain threshold. The threshold is typically set at a high percentile, such as the 95^{th} or 99^{th} percentile, to analyze only the rarest events. The Generalized Pareto Distribution (GPD) is a critical element in the field of extreme value theory, demonstrating significant utility in modeling distribution tails, which is crucial in risk management and financial analysis. This distribution is defined by shape and scale parameters, which determine the tail characteristics and are estimated using techniques such as maximum likelihood estimation (Paul & Sharma, 2021). GPD uses three main parameters:

1. ***Shape parameter (k)***: Determines whether the distribution has a longer or shorter tail. If $k>0$, extreme events are more frequent; $k=0$ indicates an exponential distribution; $k<0$ means there is a limited maximum value.
2. ***Scale parameter (σ)*****:** Defines how spread out the extreme values is above the threshold.
3. ***Location parameter (θ)*****:** Sets the starting point of the distribution, typically at zero.

The GPD demonstrates great versatility, adapting to different tail shapes depending on the value of the shape parameter, which can be positive, negative, or zero, corresponding to various categories of extreme value distributions, including Fréchet, Weibull, and Gumbel distributions (Gilli & Kellezi, 2006). GPD is particularly useful in financial scenarios, where the true distribution of returns remains undefined, with an emphasis on modeling extreme losses that occur above a significant threshold (Yao et al., 2013). This characteristic is especially useful in empirical studies, where the assumption of independent and identically distributed (i.i.d.) log returns allows the GPD to efficiently approximate the distribution of exceedances (Paul & Sharma, 2021).

The application of GPD in risk management is not without challenges, as discrepancies can arise, particularly when working with small samples. Moreover, integrating GPD with other risk metrics, such as VaR, requires careful consideration to avoid underestimating risk (Yao et al., 2013). Despite these challenges, GPD remains a powerful tool for quantifying and managing extreme risks, offering a robust framework for understanding and predicting rare, high-impact events across various disciplines, including finance and insurance.

The Value at Risk (VaR) model, based on Extreme Value Theory (EVT) and the Generalized Pareto Distribution (GPD), is used to estimate potential losses in extreme situations, focusing on rare events in the tails of the distribution that can significantly impact financial systems. The process begins by setting a threshold at a high percentile (e.g., 95th or 99th) to identify extreme events. The GPD is then applied to these values, modeling the distribution and probability of extreme events. The parameters of the GPD (shape, scale, and location) allow for precise modeling of these events, and based on this, the VaR for a given confidence level is calculated, such as 99% VaR, meaning there is only a 1% chance that losses will exceed this value. Additionally, Expected Shortfall (ES) is used as a complement to assess the average loss in worst-case scenarios. The advantage of this approach is a more accurate risk estimation compared to traditional models, which often fail to properly capture rare, fat-tail events. EVT and GPD provide flexibility in modeling different types of distribution tails, making them particularly useful for risk management in the financial sector.

The VaR value is calculated using the following formula:

$$\mathrm{VaR} = \mathrm{threshold} + \frac{\sigma}{k}\left(\left(\frac{1-q}{n/N}\right)^{-k} - 1\right) \tag{1}$$

Where: σ: the scale parameter of the GPD model, k: the shape parameter of the GPD model, q: the confidence level (e.g., 99%), n: the number of extreme events, N: the total number of observations, threshold: the threshold value above which extreme events are observed.

VaR is often calculated using models such as the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model and its derivatives, which allow for the forecasting of financial return volatility, and thus, VaR itself (Magnou, 2017; Paul & Sharma, 2021). The integration of Extreme Value Theory (EVT) with GARCH models has demonstrated improvements in the accuracy of VaR forecasts by more effectively capturing extreme market fluctuations (Magnou 2017; Paul & Sharma, 2021). However, VaR is not considered a coherent risk measure due to its failure to satisfy properties such as sub-additivity, which is why Expected Shortfall (ES) is preferred as a more coherent and reliable risk measure (Gilli & Kellezi, 2006; Magnou 2017). Despite these limitations, VaR remains an indispensable tool in financial risk management, used for setting exposure limits, calculating regulatory capital, and determining margin requirements (Paul & Sharma, 2021). The choice of model and calibration period significantly impacts the reliability of VaR estimates, with longer calibration periods generally leading to more accurate forecasts.

Expected Shortfall (ES) is a risk assessment metric that addresses several shortcomings inherent in the Value-at-Risk (VaR) method by providing insights into the magnitude of losses that may exceed the VaR threshold. Unlike VaR, which only indicates the maximum loss at a given confidence level without providing details on the severity of losses beyond that threshold, ES offers a more comprehensive perspective by estimating the expected loss that exceeds VaR (Gilli & Kellezi, 2006; Magnou, 2017). This makes ES a coherent risk metric, as it satisfies criteria such as monotonicity, sub-additivity, homogeneity, and translational invariance, criteria that VaR does not universally meet (Magnou, 2017). ES is particularly useful in the domain of financial risk management, including applications such as portfolio optimization, risk-adjusted performance evaluation, and regulatory capital risk calculations (Paul & Sharma, 2021).

Using the Generalized Pareto Distribution (GPD), ES is calculated based on extreme values above a certain threshold, where the GPD parameters (shape, scale, and location) enable the modeling of rare events in the tails of the distribution. While VaR represents the threshold below which losses occur in α-percent of cases (e.g., 99% VaR means there is a 1% chance that losses will exceed this value), ES estimates the average loss in cases where losses exceed VaR.

The value of VaR is calculated using the following formula:

$$ES = \frac{VaR}{1-k} + \frac{\sigma}{1-k} \quad (2)$$

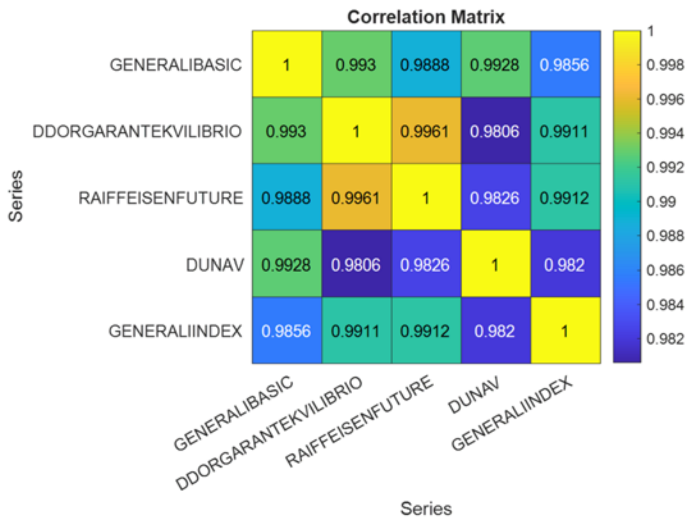
where: k: Shape parameter of the GPD, σ: Scale parameter of the GPD, VaR: The value below which α percent of the distribution lies (e.g., 99%).

Expected Shortfall (ES) is particularly useful when the GPD has a positive shape parameter, as this indicates a heavier tail in the distribution, meaning that extreme losses occur more frequently. ES provides a more accurate risk assessment for extreme losses because it not only measures the threshold like VaR, but also the expected loss when that threshold is exceeded, making it an essential metric in situations with frequent and significant extreme events.

3. Research Results

In this study, we analyze the time series of the investment unit values of five voluntary pension funds in Serbia: Dunav (DUNAV), Generali Basic (GENERALIBASIC), Generali Index (GENERALIINDEX), DDOR Garant Ekvilibrio (DDORGARANTEKVILIBRIO), and Raiffeisen Future (RAIFFEISENF), for the period from November 15, 2006 to August 12, 2024. These series were selected due to the sufficient amount of available data, enabling an adequate analysis. A detailed overview of the number of data points for each time series is provided in Table 1, offering insight into their length during the observed period. All series of investment unit values were converted into returns to achieve stationarity and to align with the standard research methodology for this type of data. All return series of the investment units are highly correlated (Figure 1).

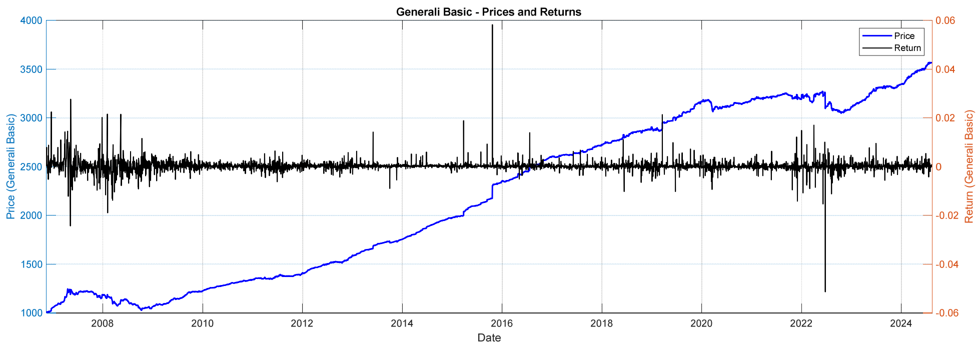
Figure 1. Correlation Matrix



Source: Author's calculations

This correlation matrix shows that the data series from the insurance table are highly correlated, with correlation coefficients above 0.98. This indicates that these series move together and exhibit very similar behavior patterns. High correlation may suggest that these series are influenced by similar factors or market conditions, which is common for financial time series within the same sector. Due to the scope of the study and the fact that all series are highly correlated, we selected the GENERALIBASIC series as a representative of the other investment unit returns. Additionally, the GENERALIBASIC series has the most data points (n=6206). Figure 2 presents the values and returns of the GENERALIBASIC investment units during the observed period.

Figure 2. Graphical representation of the movement in value and returns of the GENERALIBASIC investment units



Source: Author's calculations

Table 1 presents the results of descriptive statistics for five different data series: GENERALIB, DDOR, RAIFFEISENF, DUNAV, and GENERALIX. Each series has approximately the same number of data points, ranging from 6101 to 6206, indicating that all series are large enough for a reliable analysis. The mean values for all series are very small (ranging from 0.00011 to 0.00021). The median, which shows the central value, significantly differs from the mean, which could indicate skewness in the distribution. The standard deviation (StdDev), which measures data dispersion, varies between the series, with the smallest found in RAIFFEISENF (0.00162), indicating lower volatility, and the largest in DUNAV (0.00241), indicating higher instability. These variations are reflected in the minimum (Min) and maximum (Max) values, where DUNAV and GENERALIB show the widest range between the extreme values, while RAIFFEISENF has the narrowest range. The skewness coefficient indicates asymmetrical distribution. The GENERALIB, DUNAV, and GENERALIX series show positive skewness, meaning that the most values are concentrated on the left side of the distribution, while DDOR and RAIFFEISENF show negative skewness, meaning that the most values are on the right side. The kurtosis coefficient, which measures the "fatness" of the tails of the distribution, shows higher values indicating more extreme events.

Series such as GENERALIBASIC and DDORGARANTEKVILIBRIO have extremely high kurtosis values (176.81 and 265.6), meaning they contain many extreme values compared to a normal distribution, while GENERALIINDEX shows a lower, but still elevated, kurtosis (53.73). The Jarque-Bera test for normality confirms that none of the series follow a normal distribution. The ARCH test, which measures volatility clustering, shows that GENERALIB and DDOR do not exhibit significant evidence of heteroskedasticity, whereas the other series, particularly RAIFFEISENF and DUNAV, show pronounced volatility shifts, which may indicate changes in risk over time. However, upon closer inspection, the ARCH test varies within each series, and overall, all the observed series do not exhibit sufficient heteroskedasticity.

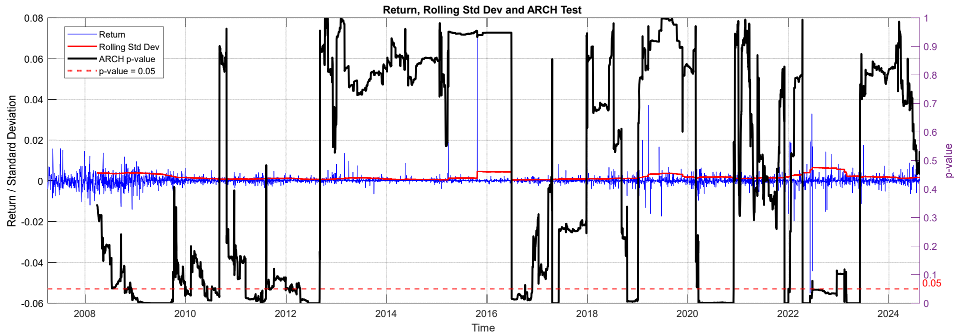
Table 1. Descriptive Statistics

	GENERAL IB	DDOR	RAIFFEIS ENF	DUNAV	GENERA LIX
Length	6206	6167	6172	6123	6101
Mean	0.00021	0.00016	0.00019	0.00020	0.00011
Median	0.00012	0.00006	0.00009	0.00010	0.00004
StdDev	0.00211	0.00206	0.00162	0.00241	0.00217
Min	-0.05141	-0.05976	-0.03377	-0.05511	-0.02036
Max	0.05824	0.05711	0.02518	0.07001	0.04433
Skewness	1.97540	-1.58610	-1.03290	2.32690	1.66470
Kurtosis	176.81000	265.60000	102.88000	214.63000	53.73000
J-B (p-value)	0.00100	0.00100	0.00100	0.00100	0.00100
ARCH (p-value)	0.09588	0.35006	0.00000	0.00000	0.00014

Source: Author's calculations

Figure 3. Graphical representation of the variability of ARCH test results for: a) GENERALIBASIC and b) DUNAV

a) GENERALIBASIC



b) DUNAV

Source: Author's calculations

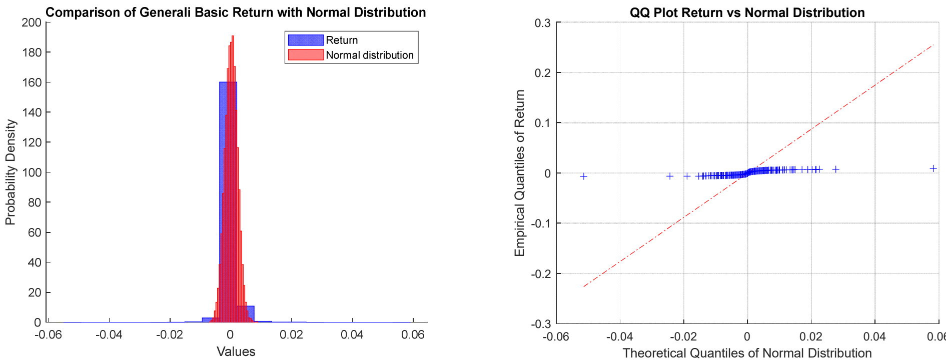
Figure 3 presents the ARCH test results (p-value) for the returns of GENERALIBASIC (a) and DUNAV (b). The charts display the return series over a specific time period, from 2008 to 2024. The returns of both investment units fluctuate around the zero value, with occasional spikes and drops. Returns are generally low in variability, with sudden changes at certain points, indicating unstable periods or market shocks. The ARCH test results (p-value) for both returns show significant volatility throughout the observed period. For instance, GENERALIBASIC exhibits heteroskedasticity during 2009 and 2012 within the last 300 days. However, after 2012, there is no evidence of heteroskedasticity in the return series. For DUNAV, the situation is similar, but the periods of heteroskedasticity are somewhat longer. Heteroskedasticity is present for almost the entire period from 2009 to 2012, during 2021, and partially during 2022 and 2023.

Although the ARCH test indicated that the series lack heteroskedasticity, the values for kurtosis and skewness suggest that the series do not follow a normal distribution. This finding implies that standard models assuming normality may not be adequate for analyzing these series. To better understand the probability distribution and extreme values in the data, we transition to a more detailed examination of the series' probability distribution. By using a QQ plot, we can visualize deviations from the normal distribution and identify any "fat tails" or asymmetric patterns, which will help us more accurately model the data distribution and better understand the risks present in the series.

Figure 4.a. represents a comparison between the histogram of the GENERALIBASIC return series and the histogram of a theoretical normal distribution with the same mean and standard deviation as the GENERALIBASIC return series. While the GENERALIBASIC return series roughly aligns with the normal distribution in the central part, there is a significant difference in the tails of distribution. The GENERALIBASIC histogram shows that the series has more extreme values at the ends (fat tails) compared to normal distribution, which

decreases more rapidly. These extreme values at the tails are characteristic of many financial series, which often exhibit "fat tails" and more extreme events than the normal distribution predicts. Figure 4.b. displays the Quantile-Quantile plot (QQ plot), a graphical tool for comparing the quantiles of the GENERALIBASIC return series with the quantiles of a normal distribution. In the central part of the plot, the points are relatively close to the diagonal line, indicating that the return series resembles a normal distribution in the central part of the distribution. However, at the tails, the points deviate significantly from the diagonal line. In the left tail (negative values) and right tail (positive values), we can observe significant deviations, indicating that the return series has more extreme values than the normal distribution predicts.

Figure 4. Distribution of GENERALIBASIC returns and graphical normality test (QQ plot).

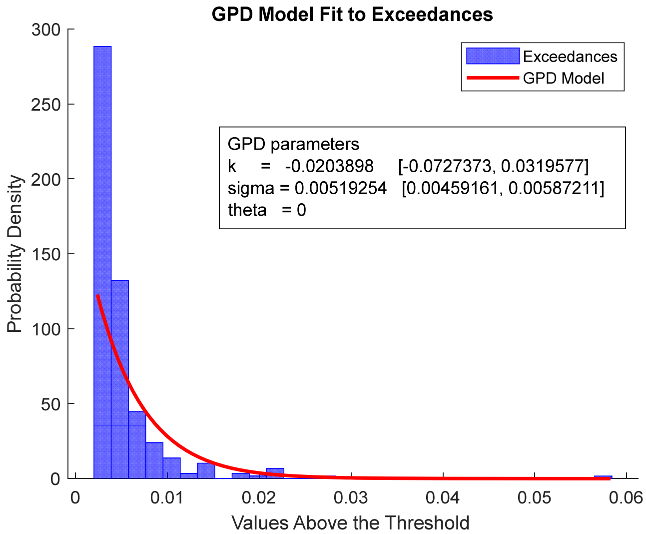


a) Distribution of returns b) QQ plot

Source: Author's calculations

After the ARCH test indicated the absence of heteroskedasticity in the time series, and after demonstrating that the returns do not follow a normal distribution, further examination of volatility requires an approach that focuses on extreme events, as standard volatility models may fail to capture rare but significant fluctuations. In this context, Extreme Value Theory (EVT) represents a natural next step, as it enables more precise modeling of the tails of the distribution and the assessment of risk associated with extreme events. EVT is particularly useful for quantifying risk in situations where events such as sudden market crashes or significant losses cannot be adequately captured by traditional approaches. By using EVT with the Generalized Pareto Distribution (GPD), we can better understand the behavior of the tails of the distribution and estimate risk indicators such as Value-at-Risk (VaR) and Expected Shortfall (ES) in the context of rare but significant events. In this direction, we tested the two most common EVT distributions: the Generalized Pareto Distribution (GPD) and the Generalized Extreme Value (GEV) distribution. For all return series, GPD proved to be the better choice (based on the AIC criterion). Figure 4 presents the histogram of extreme values from the GENERALIBASIC return series (values above the threshold) and the probability density curve of the estimated Generalized Pareto Distribution (GPD) model.

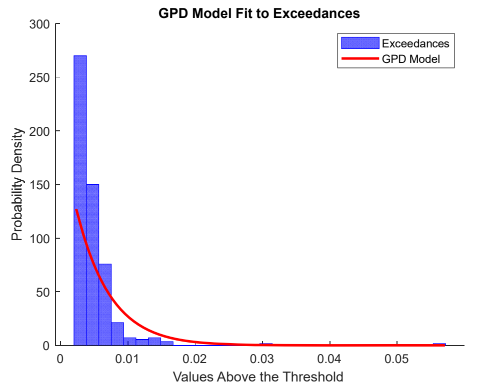
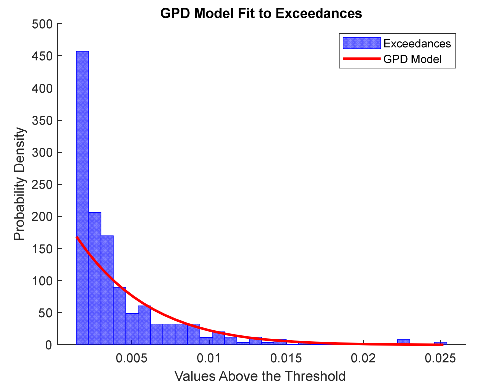
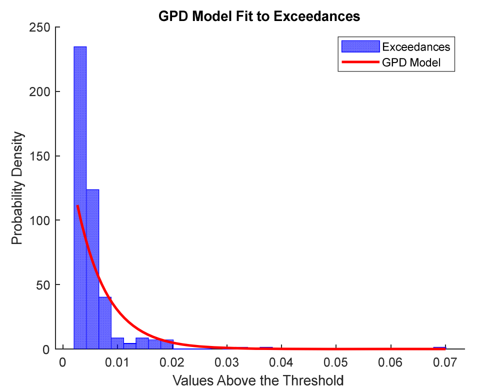
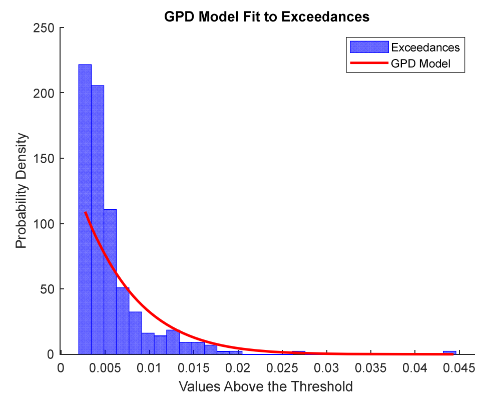
Figure 5. Probability density of the estimated Generalized Pareto Distribution (GPD) model for GENERALIBASIC returns.



Source: Author's calculations

In Figure 5, the bars represent extreme values above the threshold in the GENERALIBASIC return series (Exceedances), values that exceed a certain threshold (e.g. the 95th percentile), meaning these are rare but significant events. Most extreme values are concentrated around lower values (close to the threshold), which is expected based on the previous distribution analysis (Figure 4.a). The red line in Figure 5 represents the estimated probability density function (PDF) based on the Generalized Pareto Distribution (GPD) model that is fitted to the extreme values. This model predicts how the extreme values are distributed and should follow the shape of the histogram. In this case, we can see that the red line (GPD model) closely follows the blue bars at the beginning of the histogram, suggesting that the model accurately captures the distribution of the most extreme values. However, as we move towards the right, the red line gradually declines, indicating that extremely high values become rarer. On the graph, we can observe that for the values above 0.02, the GPD density becomes very small. This indicates that the GPD model predicts that extremely high values (above 0.02, for example) are very rare, which is typical in most financial series where extreme losses or gains are rare but potentially significant.

Table 2. Probability density and parameters of the estimated Generalized Pareto Distribution (GPD) model for the returns of investment units DDORGARANTEKVILIBRIO, RAIFFEISENFUTURE, DUNAV, and GENERALIINDEX.

DDORGARANTEKVILIBRIO	RAIFFEISENFUTURE
Generalized Pareto distribution (GPD)	Generalized Pareto distribution (GPD)
GPD parameters k = -0.0309466 [-0.0738733, 0.0119802] sigma = 0.00498352 [0.00442178, 0.00561663] theta = 0	GPD parameters k = -0.0937807 [-0.165037, -0.0225247] sigma = 0.00441555 [0.00386988, 0.00503816] theta = 0
	
DUNAV	**GENERALIINDEX**
Generalized Pareto distribution (GPD)	Generalized Pareto distribution (GPD)
GPD parameters k = 0.0006642 [-0.0556557, 0.0569841] sigma = 0.0056044 [0.00494388, 0.00635318] theta = 0	GPD parameters k = -0.0902516 [-0.136164, -0.0443396] sigma = 0.006027 [0.00534103, 0.00680108] theta = 0
	

Source: Author's calculations

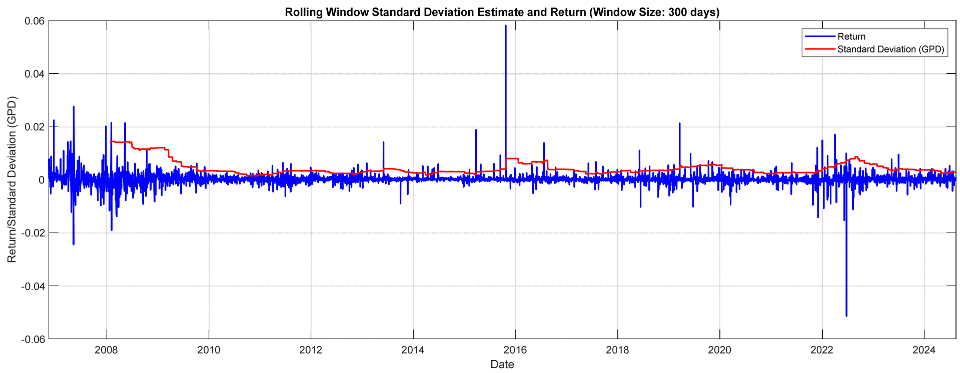
The GPD model parameters suggest that the distribution of extreme values is limited, meaning that not many extremely large values are expected (negative tail shape k = -0.0203898), and that fluctuations between the extreme values are relatively small (sigma = 0.00519254). The threshold of the GPD model (theta), above which the extreme values are analyzed, is set at 0. Based on the analysis and graphical representation, we can say that the estimated GPD model fits the histogram well, indicating that the model accurately captures the distribution of extreme values.

The same analysis was conducted for the other investment unit returns. The results of the analysis and models are presented in Table 2.

Table 2 presents the Generalized Pareto Distribution (GPD) fitting for the return series of investment units DDORGARANTEKVILIBRIO, RAIFFEISENFUTURE, DUNAV, and GENERALIINDEX. Each graph shows the distribution of values that exceed a certain threshold (blue bars) and the corresponding GPD model (red line). Each chart includes a histogram of the extreme values (so-called "exceedances") and a line representing the GPD model estimate for those values. Additionally, the parameters of the corresponding GPD model are displayed for each return series.

In all graphs, the GPD model fits well to the lower-level extreme values, while showing a declining pattern at higher values, which is characteristic of extreme values in financial time series. The thresholds are set to model only the tail of the distribution, i.e. the extreme values. The most extreme events are concentrated near the threshold, and as values increase, their frequency declines, which is clearly visible in all charts. GENERALIBASIC, DDORGARANTEKVILIBRIO, RAIFFEISENFUTURE, and GENERALIINDEX have negative k parameters, indicating bounded tails, meaning rare extreme events. DUNAV is the only series with a positive k parameter, suggesting it has an exponential tail, allowing for more extreme values. The sigma value is highest for GENERALIINDEX, indicating greater fluctuations in extreme values compared to the other series. All graphs display the expected distribution of extreme values with "thin tails," which is typical for data that do not exhibit frequent extreme fluctuations.

Figure 6. GPD model of volatility for the returns of the GENERALIBASIC investment unit

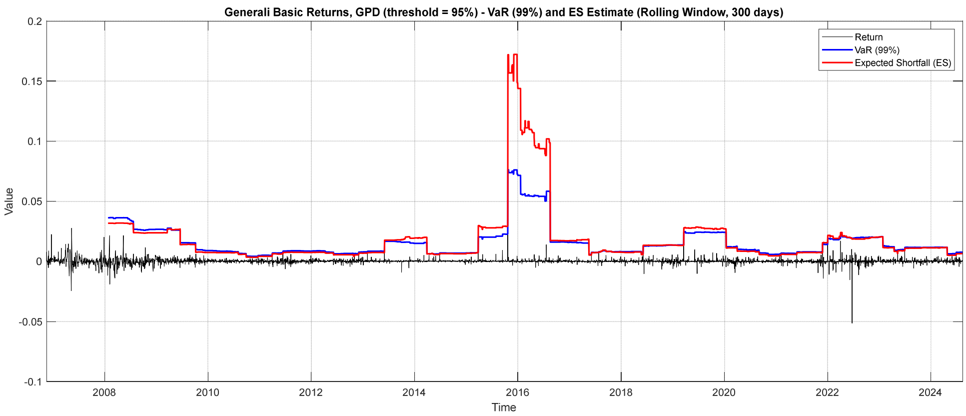


Source: Author's calculations

Figure 6 shows the time series of returns for the GENERALIBASIC investment unit (blue line) along with the estimated standard deviation (red line) based on the Generalized Pareto Distribution (GPD), using a rolling window over the last 300 days for the time period from 2008 to 2024. This chart provides a visual insight into how the volatility of the tsreturn series changes over time. It is evident that the standard deviation is higher during periods of high return fluctuations, particularly between 2008-2010, indicating greater volatility during that period. After 2010, the standard deviation decreases and remains relatively stable, with a few brief periods of elevated risk, for example, around 2016, and a slight increase in volatility in later periods. During the period from 2018 to 2024, the standard deviation is quite low, suggesting lower volatility in those years.

After successfully modeling the volatility of the observed series using the Generalized Pareto Distribution (GPD), the next logical step in risk assessment is applying this distribution to calculate key risk indicators such as Value-at-Risk (VaR) and Expected Shortfall (ES). By using the GPD model, we can more accurately quantify the risk of extreme losses, as this approach is particularly effective at capturing the "fat tails" of the distribution, which represent rare but potentially catastrophic events. VaR allows us to estimate the maximum expected loss at a given confidence level, while ES provides the average loss in cases where the VaR threshold is exceeded. These indicators, based on the GPD model, offer a more detailed insight into extreme risks in the observed series.

Slika 7. Value at Risk (VaR) and Expected Shortfall (ES) for the returns of the GENERALIBASIC investment unit



Source: Author's calculations

Figure 7 presents a risk and return analysis over time, focusing on two key risk measures: Value at Risk (VaR) and Expected Shortfall (ES), which are used to assess potential portfolio losses. This analysis covers the period from 2008 to 2024. The blue line (VaR - Value at Risk) represents VaR at a 95% probability level. VaR measures the potential loss in a portfolio that will not be exceeded in 95% of cases. On the chart, the VaR line fluctuates and tracks returns, becoming higher during the periods of increased volatility, such as in 2008 and 2016. In later periods, VaR remains relatively stable and lower, indicating less volatile market conditions.

The red line represents the Expected Shortfall (ES), another key risk indicator. ES indicates the expected loss in the event that VaR is exceeded. In other words, ES provides the average size of the loss in the worst-case scenarios (the 5% of cases that exceed the VaR threshold). This line also tracks VaR but is always slightly above it, as it measures more severe losses that surpass the given level.

4. Conclusion

Voluntary pension funds are becoming increasingly important as a savings and investment mechanism, especially in countries facing challenges in the sustainability of state pension systems. They offer flexible investment options through investment units, which allow for asset diversification and the potential for increased returns. In Serbia, two pension pillars are currently functioning, while the third pillar, voluntary pension funds, enables participants to choose funds according to their individual savings goals. Investment units serve as a key instrument of these funds, allowing participants to tailor their investment strategies based on risk profile, expected returns, and investment time horizon. Younger investors with a longer investment horizon often choose higher-risk units, while older investors tend to prefer more conservative options.

The main challenge in managing these units lies in balancing risk and return. Global economic conditions, capital market fluctuations, and regulatory changes significantly affect the unit performance, making it crucial for fund managers to continuously adjust strategies. To increase transparency, the National Bank of Serbia introduced the FONDex index to monitor the performance of investment units.

Evaluating the performance of these units through metrics like returns and volatility is a key to assessing fund success and allows participants to optimize their investments according to current market conditions and personal financial goals.

The methodology used in this study to model the volatility of financial time series is based on the application of Extreme Value Theory (EVT). The absence of heteroskedasticity in the observed series of investment unit returns prevented the use of GARCH models. Although GARCH models, especially GARCH(1,1), are often used to model volatility clustering, they could not be applied here since the series did not show significant changes in volatility over time, which is crucial for using GARCH models.

Instead, EVT was applied to adequately model extreme events in the tails of distributions. EVT uses the threshold exceedance approach, where extreme values above a certain percentile are analyzed using the Generalized Pareto Distribution

(GPD). This method allows for precise modeling of rare but significant events, a key component in risk assessment.

Value at Risk (VaR) was estimated using the GPD model, with VaR identifying the threshold below which losses will occur in 95% or 99% of cases. Additionally, Expected Shortfall (ES) was used as a complement, estimating the average loss when losses exceed the VaR threshold. The use of EVT, without relying on GARCH models, allowed for efficient modeling of extreme events in series that do not show changes in volatility, making EVT crucial for risk management in this context.

This study analyzes the time series of investment unit values from five voluntary pension funds in Serbia: Dunav (DUNAV), Generali Basic (GENERALIBASIC), Generali Index (GENERALIINDEX), DDOR Garant Ekvilibrio (DDORGARANTEKVILIBRIO), and Raiffeisen Future (RAIFFEISENF), over the period from October 27, 2008 to December 8, 2024. The selected series provide sufficient data for analysis, with all series converted into returns to achieve stationarity. The correlation matrix shows a high correlation between the series, with the correlation coefficients above 0.98, indicating that the series move together, likely under the influence of similar market conditions.

Due to the high correlation, the Generali Basic (GENERALIBASIC) series was chosen as representative for further analysis, as it has the largest number of data points (6206). Descriptive statistics show low average returns for all series, with varying degrees of volatility. RAIFFEISENF showed the lowest volatility, while DUNAV had the highest. The values for skewness and kurtosis indicate that the series do not follow a normal distribution, with several series showing significant positive skewness and high kurtosis, suggesting the presence of extreme values.

The analysis applied ARCH and Jarque-Bera tests to check for volatility and normality. The results showed that none of the series follow a normal distribution, and some series, particularly RAIFFEISENF and DUNAV, exhibited heteroskedasticity, meaning that volatility changes over time.

Furthermore, the study examined the distribution of extreme values using the Generalized Pareto Distribution (GPD) and the Generalized Extreme Value (GEV) models. Based on the Akaike Information Criterion (AIC), the GPD model proved to be the best fit for all series, successfully capturing the extreme values present in the data. Histograms of extreme values, along with the GPD models, confirmed that the distribution of these values fits well with the GPD model's predictions, particularly for lower extreme values.

Finally, the study included an analysis of Value at Risk (VaR) and Expected Shortfall (ES). VaR measures the potential maximum loss that will not be exceeded in 95% of cases, while ES estimates the average loss above the VaR threshold. The results showed the periods of increased risk, particularly between 2008-2010 and in 2016, with lower volatility in later years. The findings highlight the importance of using models like GPD, which better account for extreme events compared to normal distribution models, especially in financial data prone to rare but significant events.

References

Le, T. H. (2020). Forecasting value at risk and expected shortfall with mixed data sampling. International Journal of Forecasting, 36(4), 1362–1379. https://doi.org/10.1016/J.IJFORECAST.2020.01.008

Chavez-Demoulin, V., Embrechts, P., & Sardy, S. (2014). Extreme-quantile tracking for financial time series. Journal of Econometrics, 181(1), 44–52. https://doi.org/10.1016/J.JECONOM.2014.02.007

Haan, L., Mercadier, C., & Zhou, C. (2016). Adapting extreme value statistics to financial time series: dealing with bias and serial dependence. Finance and Stochastics, 20, 321-354. https://doi.org/10.1007/S00780-015-0287-6.

McNeil, A., & Frey, R. (2000). Estimation of tail-related risk measures for heteroscedastic financial time series: an extreme value approach. Journal of Empirical Finance, 7, 271-300. https://doi.org/10.1016/S0927-5398(00)00012-8.

Paul, S., & Sharma, P. (2021). Forecasting gains by using extreme value theory with realised GARCH filter. IIMB Management Review, 33(1), 64–70. https://doi.org/10.1016/J.IIMB.2021.03.011

Lux, T., Lux, T., Segnon, M., & Gupta, R. (2016). Forecasting crude oil price volatility and value-at-risk: Evidence from historical and recent data. Energy Economics, 56, 117–133. https://doi.org/10.1016/J.ENECO.2016.03.008

Magnou, G. (2017). An Application of Extreme Value Theory for Measuring Financial Risk in the Uruguayan Pension Fund. Central Bank of Uruguay. REVISTA DE ECONOMÍA, Vol. 24, No.1, 155-175.

Gilli, M., & Kellezi, E. (2006). An Application of Extreme Value Theory for Measuring Financial Risk. Computing in Economics and Finance, 27(2), 207–228. https://doi.org/10.1007/S10614-006-9025-7

Yao, F., Wen, H., & Luan, J. (2013). CVaR measurement and operational risk management in commercial banks according to the peak value method of extreme value theory. Mathematical and Computer Modelling, 58(1), 15–27. https://doi.org/10.1016/J.MCM.2012.07.013

Radojković, I., & Stevanović, K. (2024). Analiza rada dobrovoljnih penzijskih fondova u Srbiji u 2023. godini, International May Conference on Strategic Management, Volume XX, Issue(2), p51-55.

Kočović, J., Šulejić, P., Rakonjac, Antić T. (2010). Osiguranje, Centar za izdavačku delatnost, Ekonomski fakultet u Beogradu, Beograd

MODELIRANJE VOLATILNOSTI PRINOSA INVESTICIONE JEDINICE DOBROVOLJNIH PENZIJSKIH FONDOVA U SRBIJI

Apstrakt: Svrha ovog rada je modelovanje i analiza volatilnosti prinosa investicionih jedinica u dobrovoljnim penzijskim fondovima u Srbiji, sa fokusom na pet fondova: Dunav, Generali Basic, Generali Index, DDOR Garant Ekvilibrio i Raiffeisen Future. S obzirom na sve veći značaj dobrovoljnih penzijskih fondova, studija istražuje ulogu investicionih jedinica kao ključnog finansijskog instrumenta koji omogućava diversifikaciju i optimizaciju dugoročnih prinosa. Metodološki, studija primenjuje Teoriju ekstremnih vrednosti (EVT) koristeći Generalizovanu Pareto distribuciju (GPD) za modelovanje ekstremnih događaja u repovima distribucija, što je ključna komponenta upravljanja rizikom. ARCH test je inicijalno korišćen za procenu heteroskedastičnosti u vremenskim serijama, ali je izostanak značajnih promena volatilnosti onemogućio primenu GARCH modela. Umesto toga, EVT je primenjen kako bi se obuhvatile retke, ali značajne fluktuacije. Dodatno, Value at Risk (VaR) i Expected Shortfall (ES) su procenjeni na osnovu fitovanog GPD modela, pružajući robustniju kvantifikaciju rizika za ekstremne gubitke. Rezultati pokazuju da su sve serije prinosa visoko korelisane, sa ekstremnim vrednostima koje se pretežno javljaju u kraćim periodima. GPD modeli su uspešno uhvatili ove ekstreme, dok su VaR i ES mere ukazale na periode povećanog rizika, naročito tokom finansijskih kriza. Ovo istraživanje predstavlja originalan doprinos analizi volatilnosti investicionih jedinica, pružajući praktične uvide menadžerima fondova u balansiranju rizika i prinosa u volatilnim tržišnim uslovima.

Ključne reči: dobrovoljni penzioni fondovi, investiciona jedinica, volatilnost, Teorija ekstremnih vrednosti (EVT), Generalizovana Pareto distribucija (GPD), upravljanje rizikom

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